

# View Review

**Paper ID**

3512

**Paper Title**

Computing Welfare-Maximizing Fair Allocations of Indivisible Goods

**Track Name**

AAAI2021

**REVIEW QUESTIONS**

---

**1. {Summary} Please summarize the main claims/contributions of the paper in your own words. (Do not provide any review in this box)**

The authors study the complexity of computing allocations that are both EF1 (PROP1) and maximize the utilitarian welfare (UM). They focus on two problems: (1) Among the UM allocations, decide whether there exists one that is also EF1 (PROP1) and (2) among the EF1 (PROP1) allocations, compute one that is UM such allocation. Their analysis shows that these problems are NP-hard in general. Problem (1) is P only in the special case 2 agents. Finally, for the case of fixed number of agents, the authors design pseudo-polynomial algorithms for (1) and (2).

---

**2. {Novelty} How novel is the paper?**

Main ideas of the paper are known or incremental advances over past work

---

**3. {Soundness} Is the paper technically sound?**

The paper has minor technical flaws that are easily fixable

---

**4. {Impact} How important is the paper likely to be, considering both methodological contributions and impact on application areas?**

The paper will have low overall impact

---

**5. {Clarity} Is the paper well-organized and clearly written?**

Fair: paper is somewhat clear, but important details are missing or confusing, which hurts readability

---

**6. {Evaluation} Are claims well supported by experimental results?**

Not applicable: This paper does not have experiments

---

**7. {Resources} How impactful will this work be via sharing datasets, code and/or other resources? (It may help to consult the paper's reproducibility checklist.)**

Not applicable: no shared resources

---

**8. (Reproducibility) Would the experiments in the paper be easy to reproduce? (It may help to consult the paper's reproducibility checklist.)**

Not applicable: this research has no experimental component

---

**9. {Reasons to Accept} Please describe the paper's key strengths.**

I have no argument for accepting this paper at this moment. The paper is interesting but its marginal contribution is very little.

---

**10. {Reasons to Reject} Please describe the paper's key weaknesses.**

Evaluation: I like this work because I am from the field. However, the authors can improve the literature review. The technical quality of the paper and the presentation of the results/text can be improved. I hope you find my comments constructive. Based on them, I recommend this paper for another round of polishing.

---

**11. {Detailed Comments} Please provide other detailed comments and constructive feedback.**

Major comments:

1. From an innovative perspective, the questions studied in the paper are not new. The paper is about UM and EF, EF1, PROP, PROP1. UM and EF is stronger than PE and EF. The authors observed this: "Our results show that strengthening Pareto-efficiency to utilitarian welfare maximization...". For this reason, I find it strange that they did not discuss the below works. Is there a reason for it?

\* Bliem, Bredereck, Niedermeier. Complexity of Efficient and Envy-Free Resource Allocation: Few Agents, Resources, or Utility Levels

\*\* Bredereck, Kaczmarczyk, Knop and Niedermeier. High-Multiplicity Fair Allocation: Lenstra Empowered by N-fold Integer Programming

In fact, the authors of (\*\*) study the problem of deciding whether a E-efficient and F-fair allocation exists, where E is an efficiency measure and F is a fairness measure. They proved that the problem is fpt-tractable for the parameter  $n+u_{\max}$ . This result holds for group PE and EF1. Group PE is defined in the below work. This work is also not mentioned. Again, is there a reason for it?

\*\*\* Aleksandrov, Walsh. Group Envy Freeness and Group Pareto Efficiency in Fair Division with Indivisible Items

However, Aleksandrov and Walsh observed that n-group PE is equivalent to utilitarian efficiency, i.e. UM. It follows by this observation and Corollary 3 from "High-Multiplicity Fair Allocation..." that deciding UM and EF1 is fpt-tractable in  $n+u_{\max}$ . Hence, deciding PROP1 and UM is fpt-tractable in  $n+u_{\max}$ . By comparison, the authors show that computing/deciding these is fpt-tractable in  $n+V$ . These results are quite related if not the same. Or, am I missing something?

In any case, the relations to these existing works (and perhaps others) are not discussed which put questions of whether the results in Theorems 7.1-7.4 are new and else how these new algorithms compare to the ILP programs from the existing works?

2. From a technical perspective, the reduction of Theorem 4.1 resembles quite a lot a reduction of Freeman et al. 2019 about the hardness of EQ1 and PO. The authors reference this work. This is Theorem 1 in "Equitable allocations of indivisible goods." However, it seems to me that there are more relations between these results. For example, is there a relation between EQ1 and PO, and EF1 and UM in these reductions? It is worth a small discussion even though the main focus is on EF1 and UM. I like the reduction in Theorem 4.4. Theorems 6.1-6.3 are nice as well.

3. From a perspective of the presentation, the proofs of Theorems 4.1 and 5.1, and Theorem 4.4 and 5.4 are very similar. This makes it hard to understand why both of them are included in the main text and some other (and more interesting) results are omitted from it (Thm 7.1). Perhaps, the authors can include a discussion if these reductions have insightful differences, or else perhaps leave Thm 5.1 and 5.4 out. The same holds for Propositions 4.2 and 5.2, Corollaries 4.3 and 5.3, and 4.5 and 5.5.

For 2 agents, the authors show that the existence problems become P but the computational problems remain NP-hard. In fact, the result in Theorem 6.3 strenghtens the weak-hardness results in Corollary 4.5 (5.5) and Corollaries 4.3 (5.3) from three to 2 agents. The latter results can therefore be removed from the main text, and just discussed there.

Finally, as EF1 is stronger than PROP1, Alg 1, Alg 3 and its modification in Theorem 7.4 are sufficient. The main text may include more details about Alg 1 and the modification of Alg 3. Perhaps, I missed something but what is the advantage of having Algorithm 2 (in the main text, or at all), given that the algorithm in Theorem 7.4 also returns a PROP1 and UM allocation?

Minor comments:

- The title is "Computing Welfare-Maximizing Fair Allocations of Indivisible Goods" but the paper is about the

Social/Utilitarian welfare. Perhaps, "Utilitarian(-Maximizing) and Fair Allocations of Indivisible Goods" is a better choice because Welfare-Maximizing may refer to Egalitarian, Nash.

- The def of PROP1: " $u_i(o)$ " → " $u_i(o)$ "
- "we denote by A and B the set.." → "we denote by A and B the sets..." or "we denote by A/B the set... (remove respectively)"
- "We show that the answer is no by..." → "We show that the answer is negative by..." or "We show that the answer is "no" by..."
- "We find that while existence can be computed ..." → "We find that while existence can be decided ..."
- "...psuedo-polynomial..." → "...pseudo-polynomial..."
- "...both both ..." → "...both..."
- "... a yes instance of ..." → "... a "yes" instance.." I know what do you mean here but this is not natural language!

#### AFTER RESPONSE:

I am afraid that the authors did not address my concerns very adequately. For example, the missing references are directly related to the contribution of this work. The authors admit this in their response. From this perspective, I believe that they are essential. They also admit that the question in their focus (i.e. UM+EF1) is no longer novel because one of the existing algorithms solves it.

Also, the authors say "Reviewer #1 claimed that results for PROP1 are redundant, since the EF1 condition is stronger.". This is not true! I claim that they are redundant because their proofs are technically similar. Indeed, the proofs of 4.1, 4.2, 4.3, 4.4 and 4.5 resemble so much the ones of 5.1, 5.2, 5.3, 5.4 and 5.5 respectively. From this view, my suggestion here was that the authors could remove one of these sets of proofs but keep their statements in the main text. By doing so, they could simply have enough space to include other essential proofs (e.g. 7.1).

After the response, I can see why both Alg.2 and the one in Thm 7.4 are needed. However, the authors could include their argument in the paper. Furthermore, this raises a number of other interesting questions. For example, what is the UM loss when moving from UM-within-PROP1 to UM-within-EF1? This question could have been discussed/answered via a simple example, or a proof.

Further, it seems that there is a relation between EQ1 and PO, and EF1 and UM that is somehow hidden and not sufficiently explored. In particular, the authors admit that one of the directions in their reduction in the proof of Theorem 4.1 coincides with the corresponding direction in the reduction in the proof of Theorem 1 from Freeman et al. 2019. They say that the other direction does not. However, this suggests that the technical approach of the hardness results in this work is very similar to the one taken by Freeman et al. 2019. This is not discussed in the paper and make the technical contribution much smaller because Thm 4.1 is one of the major results in this paper and the proof of Thm 5.1 follows the same line of construction.

The motivation could also be improved. I disagree with the "motivating" setting in their response, where UM maximizes the expected sum of survivors. This is  $\sum p_i$  where  $p_i$  is the survival probability of an individual. However, it is well know that the summation may give to someone a lot and to others a few. For this reason, I find it more appealing to consider problems of EM (max. egalitarian)+EF1 (PROP1), leximin+EF1 (PROP1). In other words, how about computing the leximin solution within the set of EF1 allocations? Such an allocation will maximize the agent's utility in an EF1 allocation.

Finally, I believe that this work studies an interesting problem but it does not stand on its own as there is a solid body of literature behind the problem of interest, which is not discussed. The work further needs a better motivation of why UM, and not say the egalitarian welfare or the leximin solution, is chosen as measure of efficiency. Perhaps, leximin is a nice measure as it has some nice fairness guarantees.

#### 12. {QUESTIONS FOR THE AUTHORS} Please provide questions for authors to address during the author

**feedback period. (Please number them)**

Please see the detailed comments for questions. I would like that the authors answer these during the response phase.

---

**14. (OVERALL SCORE)**

3 - Clear reject

---

**15. (CONFIDENCE)**

Reviewer is an expert in the specific topic of the paper

---

**16. (YEARS IN AI RESEARCH) How many years have you worked in Artificial Intelligence research?**

6-10 year

---

**17. (Award) Should this paper be considered for an outstanding paper award?**

No

---

**19. I acknowledge that I have read the author's rebuttal and made whatever changes to my review where necessary.**

Agreement accepted

---